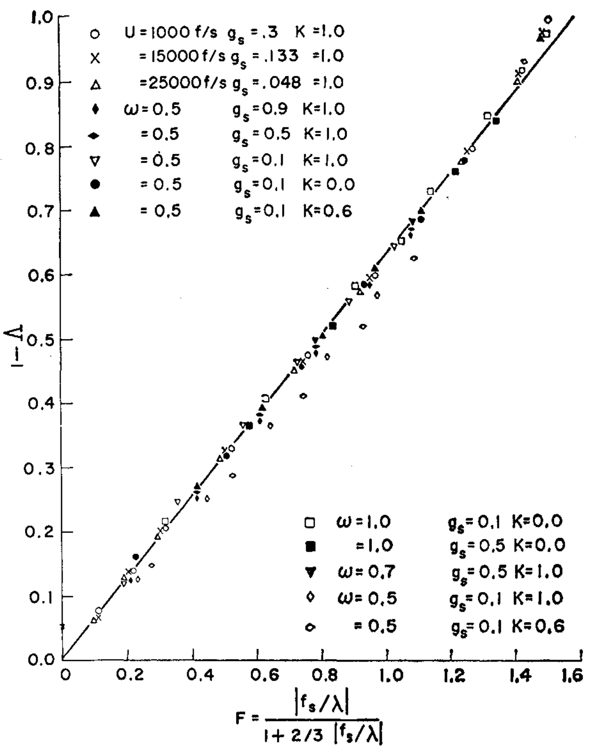

Fig. 2 Correlation of mass transfer effectiveness data for model and real gases.

($1 \leq C_s \leq 5.0$) for all shapes ($-1.0 \leq K \leq 1.0$) could be simply correlated to within 4% by

$$\lambda^*/\lambda^*_{(C=1.0, Pr=0.7)} = 0.12(C_s - 1)^{1/2} \quad (8)$$

The required λ^* data for $C = 1.0$, $Pr = 0.7$, is given in Table 1. The correlation function, Eq. (7), together with Eq. (8) and Table 1 allow the surface heat transfer to be calculated.

Data for homogeneous boundary layers, or air into air injection, have rather limited direct applicability; relatively few engineering systems, outside the field of air-breathing propulsion, are likely to use air as a coolant. More commonly an ablating material is used and the resulting mixture of gases entering the boundary layer may have properties which differ significantly from those of air. At present there are no data for foreign gas injection at three-dimensional stagnation points, but, by use of existing axisymmetric results, the applicability of the present correlation can be extended in an approximate manner. For example, Anfimov[6] showed that the effect of foreign gas injection on the conductive heat transfer to the surface can be accounted for if the injection parameter is redefined as $\phi' = (f_s/\lambda^*)(M_{air}/M_i)^{0.24}$. Thus it is tentatively proposed that for foreign gas injection into air, ϕ in Eq. (7) be replaced by ϕ'.

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Approximate Solution for Coupled Librations of an Axisymmetric Satellite in Circular Orbit

V. J. Modi* and S. K. Shrivastava†

University of British Columbia, Vancouver, B. C., Canada

Nomenclature‡

x,y,z	=	principal axes, with z along the axis of symmetry
A_i	=	functions of ψ, ψ', ϕ, ϕ'
B_f	=	aerodynamic coefficient[1]
C_1	=	constant, $\pi/2\,\{(1-K_i)/3(1+K_i)\}^{1/2}$
C_H	=	nondimensionalized Hamiltonian, $2H/I\dot{\theta}^2$
H	=	Hamiltonian
K_i	=	inertia parameter, $1 - I_{zz}/I$ where $I = I_{xx} = I_{yy} > I_{zz}$
θ	=	angular position of the satellite as measured from pericenter
θ_1^*, θ_2^*	=	phase angles
ϕ	=	rotation across the orbital plane (roll)
ψ	=	rotation in the orbital plane (pitch)
λ	=	rotation about z axis (yaw)

Introduction

ATTITUDE dynamics[2] of gravity-oriented satellites has been studied extensively in recent years.[3] The complexity of the problem has, in most cases, led to the use of numerical techniques.[1,4] However, the digital approach often fails to give an insight into the system behavior in absence of extensive computations which tend to be rather expensive.

Here, a simple approximate analytical method using the constant Hamiltonian of the system is proposed to solve a set of nonlinear, coupled equations corresponding to the general librations of a satellite.

Analysis

For a rigid, nonspinning, axisymmetric satellite negotiating a circular trajectory in the gravity gradient field with the atmospheric effect, the equations of the librational motion, with θ as the independent variable, are[1]

$$\psi'' - 2\phi'(\psi' + 1)\tan\phi + 3K_i \sin\psi\cos\psi + B_f(|\cos\psi| + C_1\sin\psi)\cos\psi/\cos^2\phi = 0 \quad (1a)$$

$$\phi'' + \{(\psi' + 1)^2 + 3K_i\cos^2\psi\}\sin\phi\cos\phi = 0 \quad (1b)$$

$$\lambda' - (1 + \psi')\sin\phi = 0 \quad (1c)$$

Recognizing that the Eqs. (1a) and (1b) do not involve λ explicitly, their solution can be undertaken independent of Eq. (1c). Multiplying them by $2\psi'\cos^2\phi$ and $2\phi'$, respectively, adding and integrating once yield the normalized

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* Professor. Member AIAA.

† Post-doctoral Fellow. Associate Member AIAA.

‡ Primes indicate differentiation with respect to θ; subscript e represents equilibrium condition.

Hamiltonian,

$$C_H = \phi'^2 + \cos^2\phi(\psi'^2 - 1 - 3K_i \cos^2\psi) + B_f\{\psi + \sin\psi\,(\cos\psi + C_1 \sin\psi)\} = \text{constant (for } |\psi| < \pi/2\text{, i.e., nontumbling motion)} \quad (2)$$

which can be rearranged as

$$\psi'^2 = [(C_H - \phi'^2)/\cos^2\phi + 1 + 3K_i - B_f(\psi + \sin\psi\,\cos\psi)/\cos^2\phi] - (3K_i + B_fC_1/\cos^2\phi)\sin^2\psi \quad (3)$$

or,

$$\phi'^2 = [C_H - \psi'^2 + 1 + 3K_i\cos^2\psi - B_f\{\psi + \sin\psi\,(\cos\psi + C_1\sin\psi)\}] - (1 + 3K_i\cos^2\psi - \psi'^2)\sin^2\phi \quad (4)$$

Putting $\psi = \bar{\psi} + \psi_e$ and $\phi = \bar{\phi} + \phi_e$, where $\psi_e = \tan^{-1}\{-B_f/(B_fC_1 + 3K_i)\}$, $\phi_e = 0$ denotes stable equilibrium con-configuration,[1] corresponding to the system's minimum potential and satisfying Routh's criteria, these relations transform to

$$\bar{\psi}'^2 = A_1 - A_2\sin^2\bar{\psi} \quad (5)$$

$$\bar{\phi}'^2 = A_3 - A_4\sin^2\bar{\phi} \quad (6)$$

where

$$A_1 = 1 + 3K_i + \{C_H - \bar{\phi}'^2 - B_f\psi_e + 3K_i\sin^2\bar{\phi}\sin^2\psi_e + (3K_i\sin^2\bar{\phi}\sin2\psi_e + B_f)\sin2\bar{\psi}/2 - B_f\bar{\psi}\}/\cos^2\bar{\phi} \quad (7a)$$

$$A_2 = 3K_i + (B_fC_1 + 6K_i\sin^2\bar{\phi}\sin^2\psi_e)/\cos^2\bar{\phi} \quad (7b)$$

$$A_3 = C_H - \bar{\psi}'^2 + 1 + 3K_i\cos^2(\bar{\psi} + \psi_e) - B_f\{\bar{\psi} + \psi_e + \sin2(\bar{\psi} + \psi_e)/2 + C_1\sin^2(\bar{\psi} + \psi_e)\} \quad (7c)$$

$$A_4 = 1 + 3K_i\cos^2(\bar{\psi} + \psi_e) - \bar{\psi}'^2 \quad (7d)$$

The solutions of Eqs. (5) and (6) can be written in terms of Jacobian elliptical functions[5] if A_1, A_2, A_3 and A_4 were constants, i.e.,

$$dA_i/d\theta = 0 \qquad i = 1,2,3,4 \quad (8)$$

For the particular case of high altitude orbits, where $B_f = \psi_e = 0$, these conditions simplify to

$$d/d\theta[(C_H - \phi'^2)/\cos^2\phi] = 0$$

$$d/d\theta[3K_i\cos^2\psi - \psi'^2] = 0$$

which are equivalent to (from Eq. 1a):

$$4\phi'\psi'(\psi' + 1)\tan\phi = 0 \quad (9)$$

thus requiring the coupling terms to be ignorable.

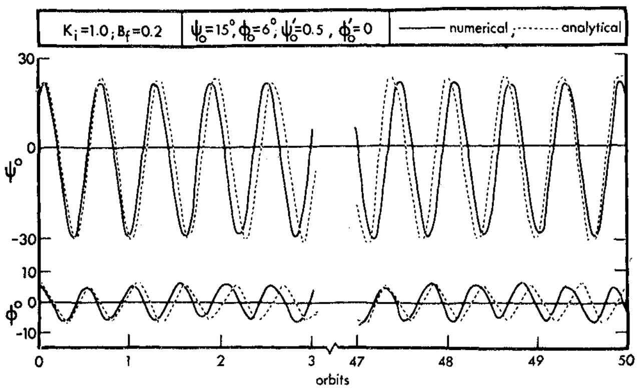

Fig. 1 Response to a predominantly planar disturbance.

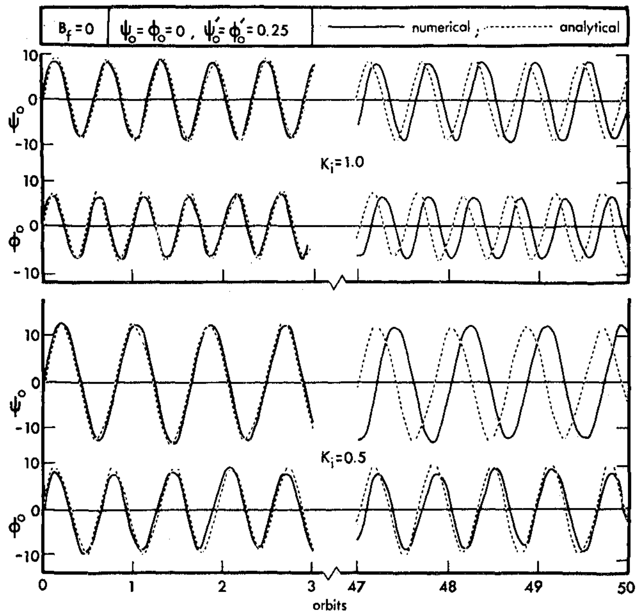

Fig. 2 Effect of satellite inertia on the librational response.

For the systems satisfying the conditions (8), the solution becomes

$$\psi = \psi_e + \sin^{-1}[(A_1/A_2)^{1/2}Sn\{(A_2)^{1/2}(\theta - \theta_1^*), A_1/A_2\}]$$
$$\phi = \sin^{-1}[(A_3/A_4)^{1/2}Sn\{(A_4)^{1/2}(\theta - \theta_2^*), A_3/A_4\}] \quad (10)$$

where $\psi(\theta_1^*) = \psi_e$, $\phi(\theta_2^*) = 0$ and A_i are determined from initial conditions.

The constraining conditions for the analytical solution appear to be rather demanding. However, it is with surprise one recognizes that, in practice, they are closely satisfied over a wide range of system parameters and initial conditions. Figures 1–3 compare, for a few typical situations, the analytical and numerical solutions over fifty orbits. For conciseness, only the initial and terminal regions are indicated. The Adams-Bashforth predictor corrector method was used to solve the exact Eqs. (1).

Discussion

From the governing Eqs. (1), it is apparent that the solution becomes exact in absence of a disturbance across the orbital plane. Any inaccuracies imparted to the solution due to a small transverse disturbance are significant only in the phase angle (Fig. 1). Even when the ϕ degree motion is of appreciable magnitude, the errors in the amplitude and frequency predictions are small for slender satellites (large K_i, Fig. 2) or higher altitude orbits (small B_f, Fig. 3). However, the planar perturbations excited by a transverse disturbance[1,4] and the minute amplitude modulations due to coupling terms are not exhibited by the analysis.

The usefulness of the solution lies in its ability to provide some appreciation as to the response characteristics of such a complex system. An identical disturbance in the two degrees of freedom excites a smaller frequency, larger amplitude motion in the ψ direction. A decrease in the satellite inertia reduces the frequency with an associated increase in the amplitude, especially for the planar motion (Fig. 2). Effect of aerodynamic torque is limited, mainly, to the shift in equilibrium position without substantially affecting amplitudes and frequencies (Fig. 3). The librational and orbital periods are of the same order of magnitude.

The solution, in general, is better than that given by the variation of parameter method[6] and was found to be acceptable (in terms of frequency and amplitude) for severe disturbances exciting motion as large as 45° (error <3%).

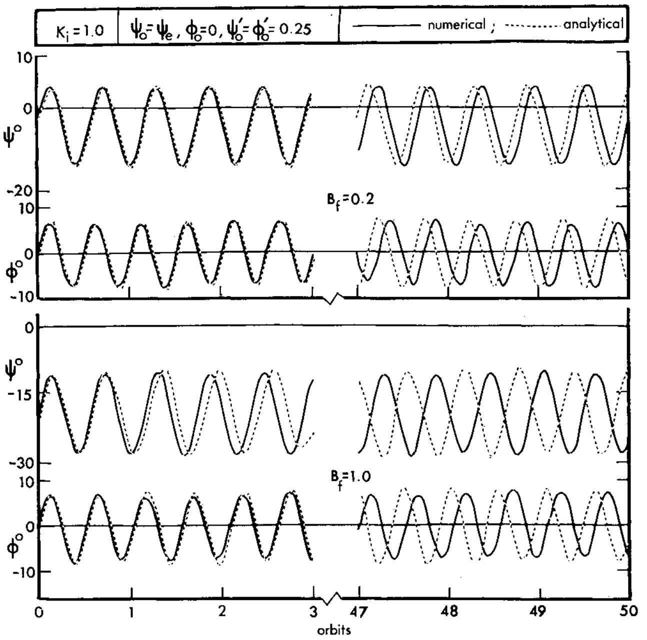

Fig. 3 Effect of atmosphere on the librational response.

It may be noted that the solution is tested here under adverse conditions. The simple analysis provides considerable insight into the physical nature of the coupled motion and appears to be adequate for preliminary design purposes.

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Variations in Secondary Mach Number and Injection Angle in Jet Interaction

Larry N. Koch* and Daniel J. Collins†

Naval Postgraduate School, Monterey, Calif.

For a period of time spanning at least the last decade, considerable interest has been displayed in the interaction of secondary gaseous flows injected into a primary supersonic stream. Several theoretical models exist which reflect the physical trends expected for varying sets of injection parameters (such as injection pressure, temperature, mass flow rate, Mach number and angle of inclination to the primary flow, etc.). The body of experimental investigation has, for the most part, concerned itself with sonic injection normal to the primary stream.

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* Lieutenant Commander, U. S. Navy.

† Professor, Department of Aeronautics. Associate Fellow AIAA.

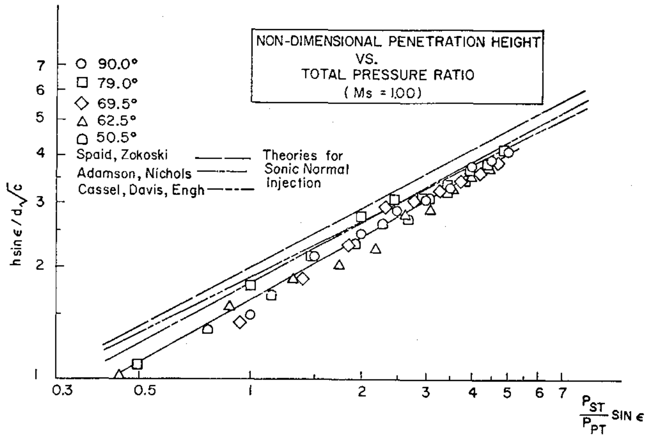

Fig. 1 Nondimensional penetration height vs total pressure ratio (M_s = 1.00).

This study was performed at the Naval Postgraduate School 4 × 4 in. Supersonic Blow-Down Wind Tunnel operating at a primary Mach number of 2.80. A flat plate model incorporating an interchangeable nozzle set and instrumented for the collection of plate surface pressure distribution was constructed and used in this investigation. Commercially available dry nitrogen, helium, and argon were injected into the main air flow at various stagnation pressures and mass flow rates through nozzles of varying Mach number and angle of inclination to the primary stream.

All nozzles have a throat diameter of 0.1000 ± 0.001 and convergence and divergence half-angles of 10°. Nozzles were constructed for injection at Mach numbers of 1.00, 1.73 ± 0.05 and 2.13 ± 0.05. For each Mach number nozzles were constructed at nominal inclination angles ϵ of 90°, 80°, 70°, 60°, 50°. Epsilon is taken to be the angle between the nozzle axis and the direction of the primary stream (or the surface of the flat plate) in a sense such that for injection not normal to the primary flow an upstream component of the secondary flow exists. The nozzles are inserted from above the model. Alignment and a positive in-place lock for securing the nozzles in position is provided by a small key bolt passing through the pylon from below. A more de-

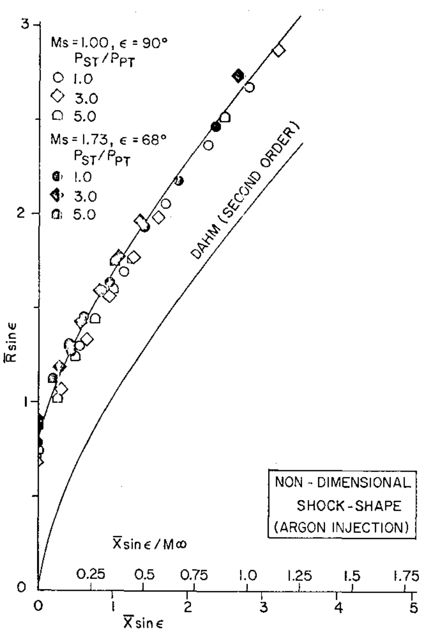

Fig. 2 Nondimensional shock-shape (argon injection).